1.

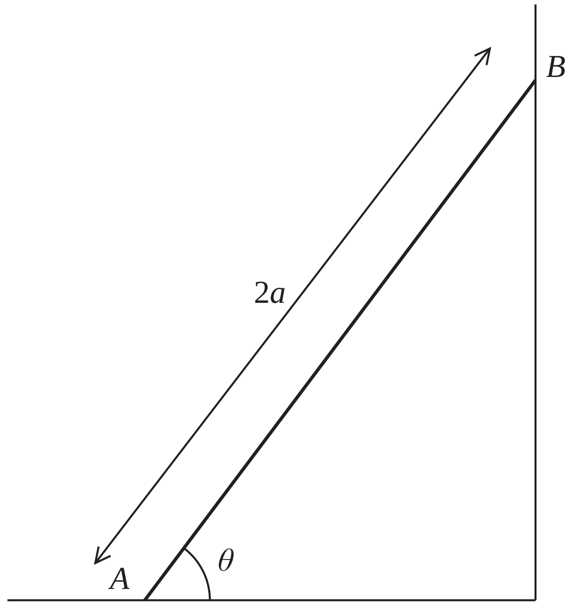


Figure 2

A beam AB has mass m and length $2a$.

The beam rests in equilibrium with A on rough horizontal ground and with B against a smooth vertical wall.

The beam is inclined to the horizontal at an angle θ, as shown in Figure 2.

The coefficient of friction between the beam and the ground is μ

The beam is modelled as a uniform rod resting in a vertical plane that is perpendicular to the wall.

Using the model,

(a) show that $\mu \geqslant \frac{1}{2}\cot\theta$

(5)

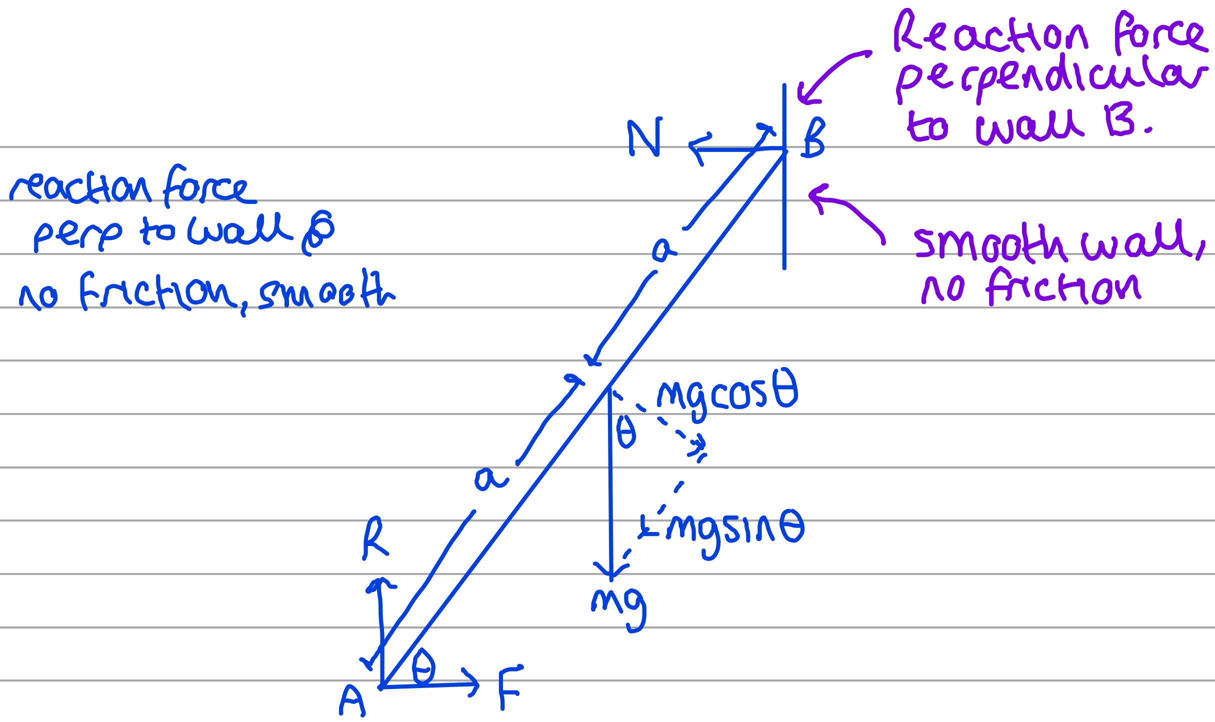
A horizontal force of magnitude kmg, where k is a constant, is now applied to the beam at A.

This force acts in a direction that is perpendicular to the wall and towards the wall.

Given that $\tan\theta = \frac{5}{4}$, $\mu = \frac{1}{2}$ and the beam is now in limiting equilibrium,

(b) use the model to find the value of k.

(5)



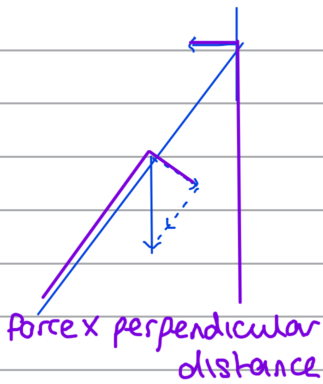
a) Beam does not slip, so $F \leq \mu R$

Method: find F and R

$R(\uparrow)$: $R = mg$ ① $R(\rightarrow)$: $F = N$

$M(A)$: ① $amg\cos\theta = 2aN\sin\theta$ ①

"Moments about A" $\Rightarrow N = \dfrac{amg\cos\theta}{2a\sin\theta}$



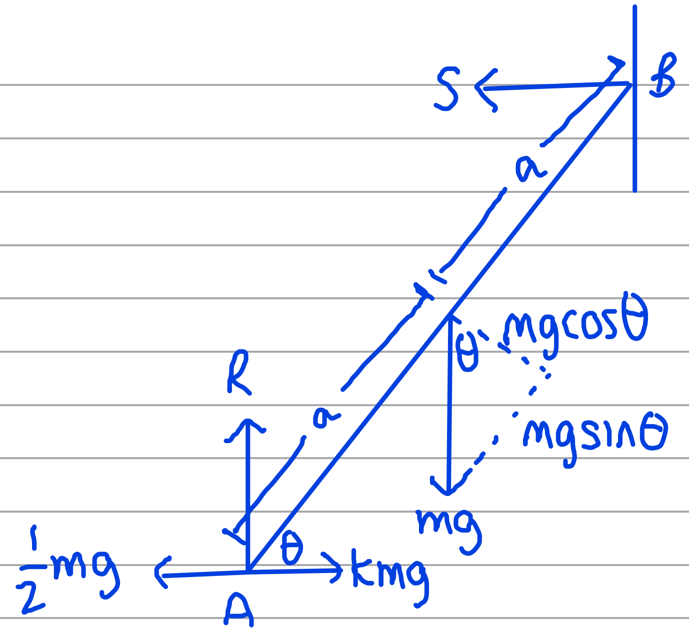
$$N = \frac{1}{2}mg\cot\theta$$

$$F = N \quad \therefore F = \frac{1}{2}mg\cot\theta$$

$$F \leq \mu R \quad \therefore \mu \geq \frac{F}{R} \quad \Rightarrow \mu \geq \frac{\frac{1}{2}mg\cot\theta}{mg} \text{ ①}$$

$$\mu \geq \frac{1}{2}\cot\theta \text{ ①}$$

b)



$\tan\theta = \frac{5}{4}$

$\therefore \cot\theta = \frac{4}{5}$

Limiting equilibrium $\therefore F = \mu R$

$\mu = \frac{1}{2} \therefore F = \frac{1}{2}mg$

Friction acts to the left as the beam is on the verge of slipping to the right

$R(\uparrow): R = mg$ (1) $R(\rightarrow): S + \frac{1}{2}mg = kmg$ (1)

$S = (k - \frac{1}{2})mg$

$M(A): amg\cos\theta = 2Sa\sin\theta$ (1)

$\Rightarrow S = \frac{1}{2}mg\cot\theta = \frac{1}{2}mg\left(\frac{4}{5}\right) = \frac{2}{5}mg$

$\therefore k - \frac{1}{2} = \frac{2}{5}$ (1)

$k = 0.9$ (1)

2.

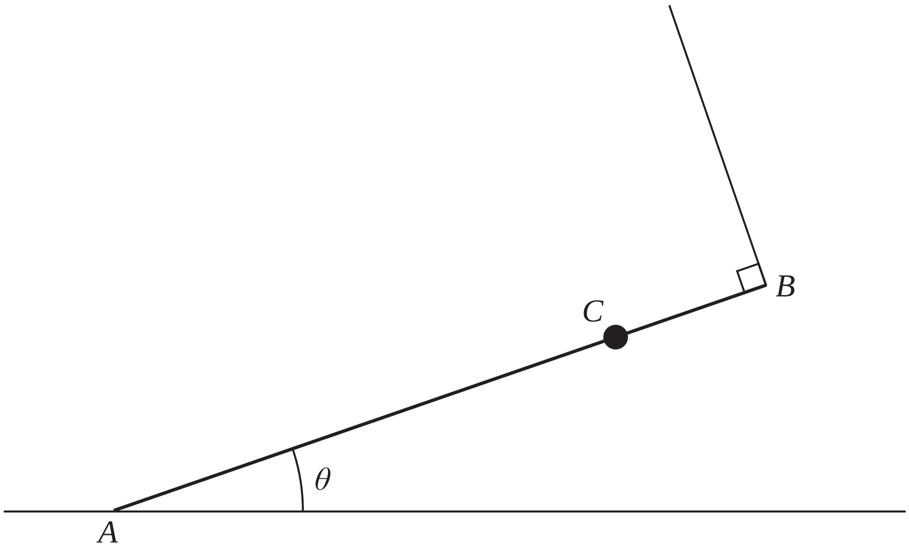


Figure 2

A uniform rod AB has mass M and length $2a$

A particle of mass $2M$ is attached to the rod at the point C, where $AC = 1.5a$

The rod rests with its end A on rough horizontal ground.

The rod is held in equilibrium at an angle θ to the ground by a light string that is attached to the end B of the rod.

The string is perpendicular to the rod, as shown in Figure 2.

(a) Explain why the frictional force acting on the rod at A acts horizontally to the right on the diagram.

(1)

The tension in the string is T

(b) Show that $T = 2Mg\cos\theta$

(3)

Given that $\cos\theta = \dfrac{3}{5}$

(c) show that the magnitude of the vertical force exerted by the ground on the rod at A is $\dfrac{57Mg}{25}$

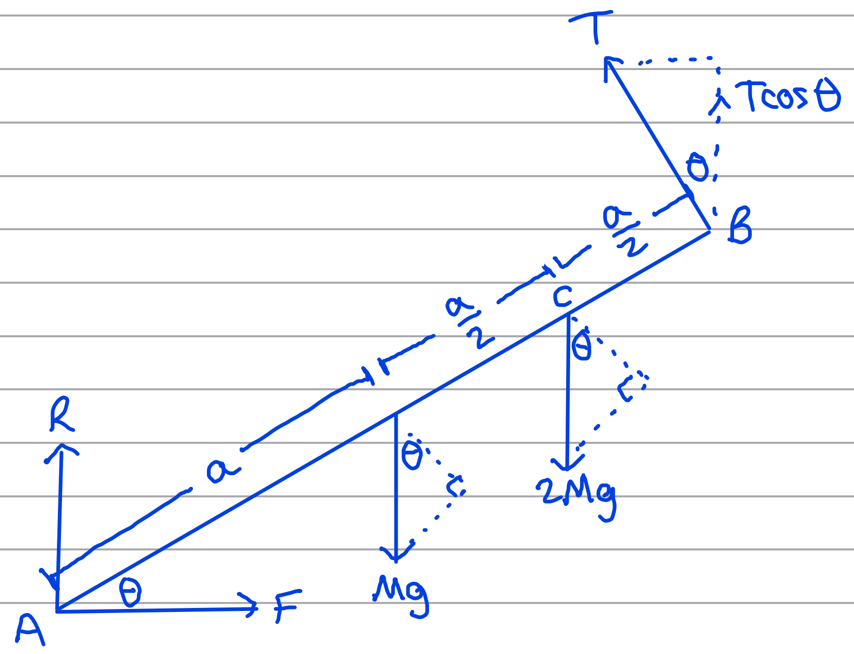
(3)

The coefficient of friction between the rod and the ground is μ

Given that the rod is in limiting equilibrium,

(d) show that $\mu = \dfrac{8}{19}$

(4)



a) The only other force that has a horizontal component is T, which acts to the left. For the rod to be in equilibrium, friction must act to the right. ①

b) $M(A): Mg \times a\cos\theta + 2Mg \times \frac{3a}{2}\cos\theta = T \times 2a$ ① ①

$$4aMg\cos\theta = 2aT$$

$$T = \frac{4aMg\cos\theta}{2a} = 2Mg\cos\theta \quad ①$$

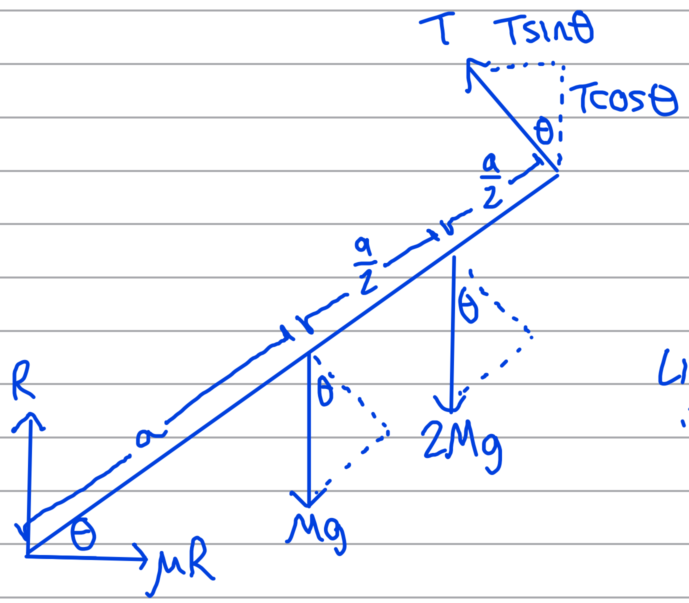
c) $\cos\theta = \frac{3}{5}$ $\quad \therefore T = 2Mg\left(\frac{3}{5}\right) = \frac{6Mg}{5}$ ①

$$R(\uparrow): R + T\cos\theta = Mg + 2Mg \quad ①$$

$$R = 3Mg - \frac{6}{5}Mg\left(\frac{3}{5}\right)$$

$$R = \frac{57Mg}{25} \quad ①$$

d)



Limiting equilibrium
$\therefore F = \mu R$ 1

$R(\rightarrow)$: $\mu R = T\sin\theta$ 1 1

sub in $R = \dfrac{57Mg}{25}$, $T = \dfrac{6Mg}{5}$, $\sin\theta = \dfrac{4}{5}$

$$\mu = \frac{T\sin\theta}{R} = \frac{\frac{6}{5}Mg \times \frac{4}{5}}{\frac{57Mg}{25}} = \frac{8}{19}$$ 1

3.

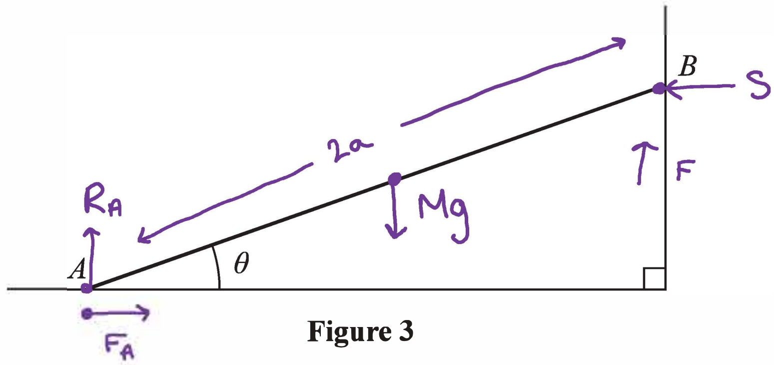


Figure 3

A rod AB has mass M and length $2a$.

The rod has its end A on rough horizontal ground and its end B against a smooth vertical wall.

The rod makes an angle θ with the ground, as shown in Figure 3.

The rod is at rest in limiting equilibrium.

(a) State the direction (left or right on Figure 3 above) of the frictional force acting on the rod at A. **Give a reason for your answer.**

(1)

The magnitude of the normal reaction of the wall on the rod at B is S.

In an initial model, the rod is modelled as being **uniform**.

Use this initial model to answer parts (b), (c) and (d).

(b) By taking moments about A, show that

$$S = \frac{1}{2} Mg \cot\theta$$

(3)

The coefficient of friction between the rod and the ground is μ

Given that $\tan\theta = \frac{3}{4}$

(c) find the value of μ

(5)

(d) find, in terms of M and g, the magnitude of the resultant force acting on the rod at A.

(3)

In a new model, the rod is modelled as being **non-uniform**, with its centre of mass closer to B than it is to A.

A new value for S is calculated using this new model, with $\tan\theta = \frac{3}{4}$

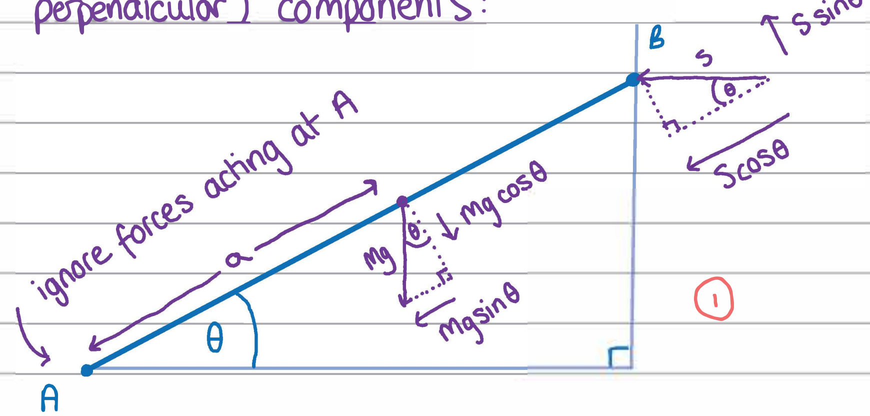
(e) State whether this new value for S is larger, smaller or equal to the value that S would take using the initial model. **Give a reason for your answer.**

(1)

(a) Frictional force at A acts right because it must oppose the normal reaction at B, which acts left. 1

(b) Calculate the horizontal and vertical (or parallel and perpendicular) components:

moment = force × distance from point to force



$$a Mg\cos\theta = 2aS\sin\theta$$ 1

÷2a

$$\frac{a}{2a} Mg\cos\theta = S\sin\theta$$

÷ sinθ

$$\frac{a}{2a} Mg \times \frac{\cos\theta}{\sin\theta} = S$$

$\cot = \frac{1}{\tan}$, $\tan = \frac{\sin}{\cos}$ ⟹ $\cot = \frac{\cos}{\sin}$

$$\frac{1}{2} Mg \times \cot\theta = S$$ 1

(c) Resolving vertically : $R = mg$ 1

Resolving horizontally: $F = S$ 1

the system is in equilibrium, so vertical and horizontal forces must be equal.

$F = \mu R \Rightarrow \mu R = S \Rightarrow \mu mg = S$ 1

$\frac{1}{2} Mg \times \cot\theta = S$ ← from part (b)

$\frac{1}{2}Mg \times \frac{4}{3} = \mu Mg$ ① $\longleftarrow$ $\tan\theta = \frac{3}{4} \Rightarrow \frac{1}{\tan\theta} = \frac{4}{3}$

$\div Mg$

$\frac{1}{2} \times \frac{4}{3} = \mu$

$\mu = \frac{2}{3}$ ①

(d) Forces acting on A: R = normal reaction = Mg
$F = \mu R = \frac{2}{3}Mg$

Magnitude $= \sqrt{F^2 + R^2}$ ①

$= \sqrt{\left(\frac{2}{3}Mg\right)^2 + (Mg)^2}$ ①

$= \sqrt{\frac{4}{9}M^2g^2 + M^2g^2}$

$= \sqrt{\frac{13}{9}M^2g^2}$

$= \frac{1}{3}Mg\sqrt{13}$ ①

(e) New value of S would be larger because the moment of the weight about A would be larger. ①